APPROXIMATE METHOD FOR DETERMINING THE MOMENTUM IN A WEDGE-SHAPED NOTCH WITH THE INSTANTANEOUS DISPERSION OF A THIN LAYER OF MATTER FROM ITS SURFACE

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UDC 662.215.2:532.522

The article proposes a geometric method, based on the superposition of elementary momenta with the repeated reflection of particles scattered from the surface of the notch. The results of the calculation are in good agreement with experiment.

1. In a number of cases, it is required to determine the pressure pulse acting on some surface, from a thin layer of matter scattered from the surface in the form of fragments and in a gaseous state; for example, from an explosive with explosive stamping. This problem is usually described by the equations of gasdynamics, which in a two-dimensional region are solved in an electronic computer. However, an analysis of existing solutions [1-3] shows the possibility of a simplified problem if the aim of the solution is merely determination of the momentum.

We assume that for a flat surface the momentum is unknown. Under analogous conditions for a wedge-shaped notch it can be found approximately in the following manner.

An element of area of the explosive can be regarded as a particle with the momentum

$$dr = i\,ds \tag{1.1}$$

in the direction of an external normal to the surface of the notch, where i is the known intensity of the impulse. If the notch is symmetrical (Fig. 1), then the motion of every particle is limited by the plane of symmetry yz and the starting surface; with a collision with the starting surface, the vector of the momentum of the particle can change in value and direction, and the barrier receives a corresponding impulse. We shall assume the collision process to be elastic and, neglecting the nonlinear problem, we shall admit the possibility of elementary impulses. In such a statement, the problem is solved purely geometrically.

Particles scattering from the surface AB at the point C arrive at the plane of symmetry at the angle $\beta_1 = \pi/2 - \alpha$. Since, with an elastic collision, the angle of reflection from the plane of symmetry and from the face of the notch is equal to the angle of incidence (the mass of the body having the notch is assumed to be infinitely great, considerably greater than the mass of the explosive), the particle strikes the starting surface at the point E at an angle $\gamma_1 = \pi/2 - 2\alpha$, and then strikes the point F at an angle of $\beta_2 = \pi/2 - 3\alpha$, and the point G at the angle $\gamma_2 = \pi/2 - 4\alpha$. Obviously, the k-th collision with and reflection from the starting surface is at an angle $\gamma_k = \pi/2 - 2k\alpha$. The maximal possible number of collisions is determined from the condition $\gamma_m = \pi/2 - 2m\alpha \geq 0$, whence

$$m = [\pi/4\alpha] \tag{1.2}$$

(here, the square brackets denote the whole part of the number included in it).

We note that collisions between particles and the starting surface are possible if $2\alpha < \pi/2$; in the contrary case, particles from the plane of symmetry immediately depart to infinity, and the mutual effect of the faces of the notch is eliminated.

Chelyabinsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 159-163, September-October, 1974. Original article submitted June 29, 1973.

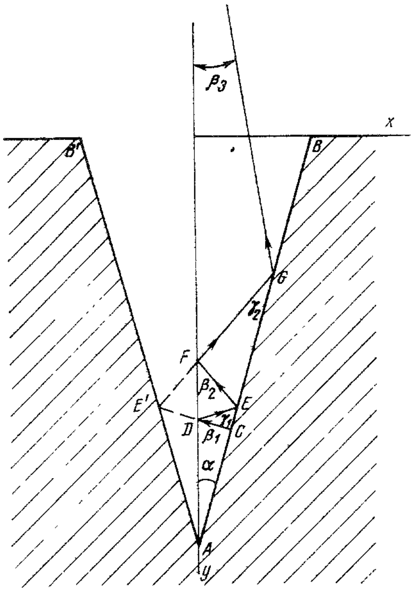

Fig. 1

After each collision, the normal component of the vector of the momentum of a particle changes sign and, consequently, the barrier receives the impulse 2 cos $2k\alpha dr$.

The total impulse from a particle reflected m times is

$$(dr)_m = \left(1 + 2\sum_{k=1}^{m} \cos 2k\alpha\right) dr \,. \qquad (1.3)$$

It is well known that [4]

$$\sum_{k=1}^{m} \cos 2k\alpha = \frac{\sin(2m+1)\alpha}{2\sin\alpha} - \frac{1}{2} \,. \qquad (1.4)$$

therefore,

$$(dr)_m = i\,\frac{\sin(2m+1)\alpha}{\sin\alpha}\, ds \,. \qquad (1.5)$$

The projection of the impulse (1.5) on the vertical direction (along the y axis) is

$$(dq)_m = i\sin(2m+1)\alpha\; ds \,. \qquad (1.6)$$

This result can be obtained in a different way, if it is taken into consideration that the angle formed by the vector of the momentum of a particle with the plane yz, with the last reflection from the face, $\beta^{m+1} = \pi/2 - (2m+1)\alpha$.

We set $2\alpha < \pi/2$ and find the limiting distance AC from the vertex of the notch, with which a particle scattered from point C strikes the starting surface mtimes. From Δ ACD, AED, and AEF, we find

$$\begin{aligned} AD &= AC/\cos\alpha \\ AE &= AD\frac{\sin(\pi/2-\alpha)}{\sin(\pi/2-2\alpha)} = AC/\cos 2\alpha \,. \\ AF &= AE\frac{\sin(\pi/2-2\alpha)}{\sin(\pi/2-3\alpha)} = AC/\cos 3\alpha \,. \end{aligned} \qquad (1.7)$$

Denoting the distance of the k-th point of collision between a particle and the face by b_k, we obtain

$$b_k = AC\,/\cos 2k\alpha \,. \qquad (1.8)$$

Since b_k cannot exceed the width of the face l, the limiting distance AC, at which a particle is reflected k times, is

$$c_k = l\cos 2k\alpha \,. \qquad (1.9)$$

In practice, the most important case is the case where the layer of explosive has a constant thickness over the whole notch. Under these circumstances, i = const, and the total vertical momentum of a particle reflected m times will be (per unit length of the notch)

$$Q_m = 2\int_0^{c_m} (dq)_m = 2il\sin(2m+1)\alpha\cos 2m\alpha \,. \qquad (1.10)$$

The total momentum of particles reflected k times is

$$Q_k = 2\int_{c_{k+1}}^{c_k} (dq)_k = 4il\sin^2(2k+1)\alpha\sin\alpha \qquad (1.11)$$

$$k = 0, 1, 2, \ldots, m-1 \,.$$

The total momentum from all the particles is

$$Q = 2il\left[2\sin\alpha\sum_{k=0}^{m-1}\sin^2(2k+1)\alpha + \sin(2m+1)\alpha\cos 2m\alpha\right] . \qquad (1.12)$$

Going over from trigonometric functions to indicating functions, and using the formula of the sum of a geometric progression, after transformations we obtain

$$Q = il\sin\alpha\left[2m-1+\frac{\sin(2m+1)2\alpha}{\sin 2\alpha}\right] . \qquad (1.13)$$

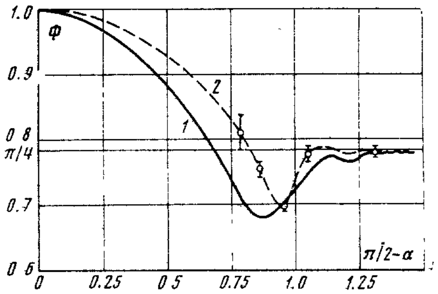

Fig. 2

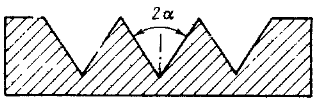

Fig. 3

The ratio of Q to the impulse from a layer of explosive with a breadth $2l$ on a flat surface,

$$\Phi = \frac{\sin\alpha}{2}\left[2m - 1 + \frac{\sin(2m-1)2\alpha}{\sin 2\alpha}\right], \tag{1.14}$$

characterizes the effect of the profile of the notch.

A curve of the function Φ is shown in Fig. 2 (1 is a calculated curve, 2 an experimental curve).

At points where $\pi/4\alpha$ is a whole number, Φ is continuous along with its derivative; in the intervals between these points, Φ has two extrema: a maximum and a minimum; Φ takes on its least value with $\alpha = 0.707$, and it is equal to 0.681. When α decreases to zero, Φ approaches the asymptotic value $\pi/4$.

2. Let us assume that a thin layer of explosive is located at only one face of the notch.

Then a particle from point C strikes point E' of the opposite face, symmetrical with respect to E, and is then reflected back to point G of the starting surface (Fig. 1). Both faces receive the same vertical impulse as a single face with a symmetrical arrangement of the explosive, i.e., Q/2.

In addition to the vertical impulse, there is also a horizontal impulse (along the x axis).

A particle reflected m times from both faces gives thefollowing impulse in a horizontal direction:

$$(dt)_m = i\cos(2m+1)\alpha\, ds \tag{2.1}$$

The total impulse of particles reflected m times is

$$T_m = \int_0^m (dt)_m = il\cos(2m+1)\alpha\cos 2m\alpha\ ; \tag{2.2}$$

k times,

$$T_k = \int_{k+1}^{k} (dt)_k = il\sin(2k+1)2\alpha\sin\alpha \tag{2.3}$$
$$k = 0, 1, 2, \ldots, m-1\ .$$

The total horizontal impulse is

$$T = il\sin\alpha\left[\sum_{k=0}^{m-1}\sin(2k+1)2\alpha - \frac{\cos(2m+1)\alpha\cos 2m\alpha}{\sin\alpha}\right]. \tag{2.4}$$

After transformations, analogously to (1.13), we find

$$T = il\sin\alpha\left[\frac{1}{2\operatorname{tg}\alpha} - \frac{\cos^2(2m-1)\alpha}{\sin 2\alpha}\right]. \tag{2.5}$$

A layer of explosive with a breadth l at an inclined surface, coinciding with the face of the notch, gives the following impulse in a horizontal direction:

$$T = il\cos\alpha\ . \tag{2.6}$$

The ratio of (2.5) and (2.6),

$$\Psi = \frac{1}{2}\left\{1 - \left[\frac{\cos(2m-1)\alpha}{\cos\alpha}\right]^2\right\}, \tag{2.7}$$

characterizes the effect of the profile of the notch on the value of the horizontal impulse.

With m = 0, $\Psi = 1$ (there is no mutual effect of the faces), when α decreases to zero, Ψ approaches a minimal value equal to 0.5.

3. Let the width of the layer σ be less than the width of a face of the notch, for example,

$$c_n \leqslant \sigma < c_{n-1} \quad (n \leqslant m+1) ,$$

i.e.,

$$\cos 2n\alpha \leqslant \sigma / l < \cos 2 (n - 1)\alpha . \tag{3.1}$$

This is possible under the condition

$$n = 1 + \left[\frac{1}{2\alpha} \arccos \frac{\sigma}{l}\right] . \tag{3.2}$$

In such a case, there are no particles having n − 2 reflections, and only some having n − 1 reflections. Consequently, the corresponding impulses must be deducted from the total vertical and horizontal impulses.

4. Experimental investigations were made on samples with a ribbed surface, made of various materials (Fig. 3). A thin layer of high explosive was deposited uniformly on the surface of the sample. The sample with a detonating device was fastened to a ballistic pendulum, from whose deviation the value of the impulse was determined. The impulse from the detonating device was determined separately. The ribbed samples were compared with flat samples made of the same material. The aim of the investigations was to determine the profile of the notch (the angle α) with which the impulse arriving at unit weight of explosive is minimal.

The values of Φ obtained experimentally from the character of the dependence on α coincide with the calculated values, somewhat exceeding them in magnitude. As an illustration, Fig. 2 gives experimental values of Φ (the mean values from five experiments and their scatter), obtained on a standard PS1-350 foam plastic.

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